PREDICTIVE ANGULAR POSITION CONTROL USING EVENT-TIME ESTIMATION AND LATERALLY POLARIZED PERMANENT MAGNET

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Abstract. High-precision angular positioning in compact rotating systems is often limited by mechanical inertia, control latency, and sensor response time, leading to overshoot when conventional feedback-based stopping strategies are applied. This work presents a predictive angular position measurement and control approach based on a laterally polarized miniature permanent magnet mounted on the shaft end, combined with an event-time estimation algorithm for anticipatory stopping. The lateral magnet polarization generates a distinct and repeatable magnetic field signature as a function of angular displacement, enabling high-resolution contactless angle sensing within a minimal axial footprint. Instead of relying solely on instantaneous angle feedback, the proposed method estimates the future time at which a predefined angular threshold will be reached by analysing the temporal evolution of magnetic signal features. The predicted event time is then used to trigger an early stop command, compensating for rotational inertia and actuator dynamics. Experimental validation demonstrated that the predictive stopping strategy reduced angular overshoot by up to 85% compared to conventional reactive stopping and improved final positioning accuracy to better than ± 0.5°. Stable operation was maintained under non-periodic sensor noise conditions, while the average rotational speed closely followed the reference trajectory without oscillatory behaviour. The proposed method enabled repeatable overshoot-free stopping even in the presence of delayed inertial response. The results confirm that lateral magnet polarization improves angular sensitivity and repeatability in compact shaft-end configurations, while event-time estimation provides a computationally efficient predictive control strategy suitable for embedded motion-control applications with limited processing resources.

Keywords: predictive angular control, encoder, shaft end angle sensing, event-time estimation, inertial compensation.

Introduction

High-precision angular positioning is a fundamental requirement in robotics, micro-actuators, optical alignment systems, precision valves, and embedded servo drives. In compact rotational systems, non-negligible inertia, friction variability, and actuator latency significantly affect stopping accuracy. Overshoot caused by stored kinetic energy is often the dominant source of positioning error when the control strategy is purely reactive [1].

Classical angular position control is typically implemented using proportional-integral-derivative (PID) controllers due to their structural simplicity and industrial maturity [1; 2]. PID controllers can be tuned to reduce overshoot and achieve acceptable settling times [3]; however, their performance is sensitive to parameter variations and modelling uncertainty [4]. When system inertia or load torque changes, retuning is often required. Furthermore, since PID control reacts to instantaneous position error, braking action is initiated only after the trajectory has evolved close to or beyond the target angle, which limits achievable stopping precision under high-speed or low-damping conditions [5; 6].

Feedforward inertia compensation and two-degree-of-freedom control structures improve transient response by incorporating model-based elements [7]. However, these approaches depend on accurate parameter identification and still operate within a feedback-error framework rather than explicitly predicting the exact time at which a predefined angular position will be reached.

Model Predictive Control (MPC) extends control performance by solving a constrained optimization problem over a finite prediction horizon [8]. MPC can account for inertia, actuator limits, and state constraints, making it attractive for high-performance motion systems. Nevertheless, its computational burden and modelling requirements may be excessive for low-cost embedded controllers used in miniature drive systems. Moreover, MPC is typically formulated in discretized time steps and does not inherently focus on deterministic estimation of a single critical event, such as the precise stopping instant.

Accurate sensing is equally critical for precise angular control. Optical encoders provide high resolution but increase system size and cost. Magnetic sensing using permanent magnets offers compactness, robustness, and tolerance to environmental contamination [9]. The polarization orientation of the magnet strongly influences the angular field distribution [10; 11]. While axially polarized magnets are common in end-of-shaft configurations, laterally polarized miniature permanent magnets can produce a steeper and more distinguishable angular field gradient in compact geometries [12; 13], improving sensitivity and repeatability within a defined operating range.

This work extends an event-time estimation concept previously developed for predictive coordination of renewable and non-renewable energy sources in energy-intensive processes [14]. In that application, the algorithm estimated the future time at which a threshold state would occur, enabling anticipatory control actions aligned with the predicted event. Here, the same mathematical principle is applied to rotational motion control: the controller continuously estimates the future time at which the shaft will reach a predefined angular position, based on the measured angle, angular velocity, and known or identified inertia. Instead of reacting to present position error, the proposed method formulates stopping as a predicted event problem. By issuing a braking or stop command at a calculated earlier instant, the algorithm compensates for inertia and system latency, aiming to achieve zero overshoot at the desired final angle [15; 16]. This event-time-based predictive stopping provides a computationally efficient intermediate solution between classical PID control and full MPC, particularly suitable for embedded motion control systems with strict space and processing constraints.

Materials and methods

A. Rotational Dynamics Model

The rotational motion of a rigid shaft driven by an actuator can be described by the classical equation of motion:

$$J\dot{\omega}(t) = T_m(t) - T_L(t) - B\omega(t), \tag{1}$$

where J – total rotational inertia;
ω – angular velocity;
T_m – motor torque;
T_L – load torque;
B – viscous friction coefficient [1; 2].

The angular position $\Theta(t)$ is obtained as:

$$\dot{\Theta}(t_s) = \omega(t) \;\text{ or }\; \Theta(t_s) = \int \omega(t)dt. \tag{2}$$

For stopping at a predefined target angle θ_{ref}, the control objective is:

$$\Theta(t_s) = \Theta_{ref},\; \omega(t_s) = 0, \tag{3}$$

where t_s – denotes the stopping time.

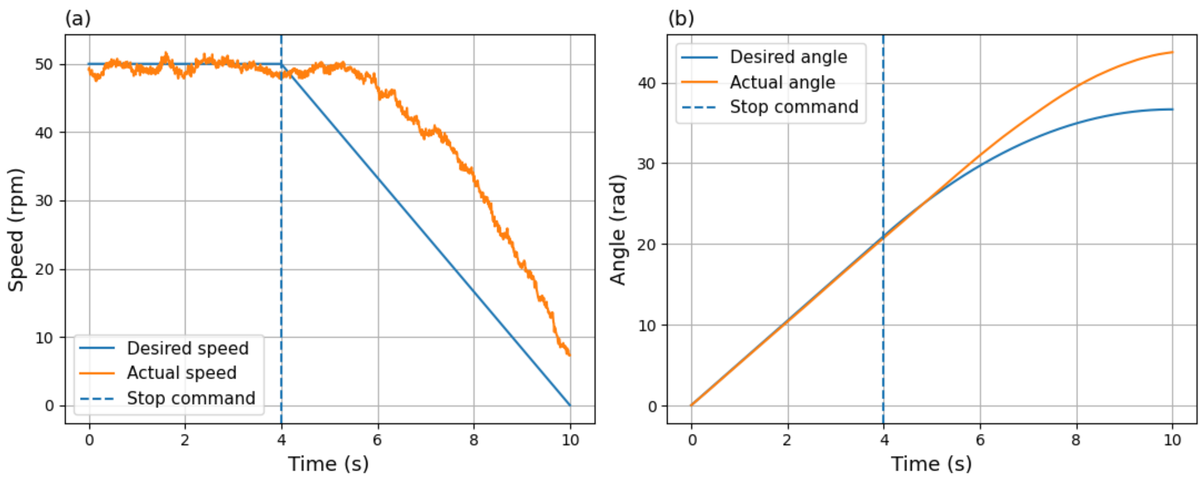

Fig. 1. **Graphs of shaft rotation speed and turning angle (simulated data):** a – conventional stopping without prediction – the stop command is issued at the reference transition point, but the actual speed response is delayed due to inertia; b – resulting angular trajectory exhibiting overshoot, highlighting the limitation of reactive control strategies in high-inertia systems

Figure 1 illustrates the limitations of conventional stopping strategies. When the stop command is issued at the reference transition point, the actual rotational speed exhibits a delayed response due to inertia, resulting in overshoot in the angular position. This effect becomes particularly significant in systems with high inertia or limited braking torque.

B. Overshoot Due to Inertia

When a stop command is issued at time t_0, the shaft still possesses kinetic energy

$$E_k = \frac{1}{2} J \omega^2 (t_0). \tag{4}$$

Even if the driving torque is removed, the system continues rotating until this energy is dissipated. Assuming approximately constant deceleration during braking, the additional angular displacement after the stop command can be estimated as:

$$\Delta\Theta \approx \frac{\omega^2(t_0)}{2|\varepsilon|}, \tag{5}$$

where ε – effective braking deceleration.

This inertial displacement is the primary source of positioning error in conventional control systems. Classical PID control can reduce this error but cannot eliminate it completely, since the control action is based on present error rather than predicted future system behaviour [2].

C. Event-Time Formulation of the Stopping Problem

To overcome this limitation, the stopping problem can be reformulated as a future event prediction task. Instead of minimizing the instantaneous position error

$$e(t) = \Theta_{ref} - \Theta(t), \tag{6}$$

the proposed approach focuses on predicting the time at which the target position will be reached.

The target event is defined as:

$$\Theta(t_e) = \Theta_{ref}. \tag{7}$$

The objective is to estimate the future event time t_e based on current measurements $\Theta(t)$, $\omega(t)$ and possibly$\dot{\omega}(t)$.

Assuming locally constant angular acceleration ε, the short-term angular trajectory can be approximated as:

$$\Theta(t+\tau) = \Theta(t) + \omega(t)\tau + \frac{1}{2}\varepsilon\tau^2. \tag{8}$$

Solving this expression for τ under the condition

$$\Theta(t+\tau) = \Theta_{ref} \tag{9}$$

yields an estimate of the time-to-event τ_e, and the predicted stopping time is

$$t_e = t + \tau_e. \tag{10}$$

The stop command is then issued at an earlier time $t_c < t_e$, considering actuator delay and braking dynamics, such that the shaft comes to rest exactly at the target angle.

This approach differs fundamentally from classical feedback control. Instead of reacting to instantaneous error, it anticipates the future system state and minimizes the predicted terminal error.

D. Relation to Predictive and Optimal Control

Model Predictive Control (MPC) computes optimal trajectories over a finite horizon using a dynamic model and constraints [8]. While MPC can achieve high precision, it requires solving an optimization problem at each sampling step.

The proposed event-time estimator can be interpreted as a reduced-form predictive control strategy:

- it uses a local dynamic model (inertia-based motion equation);
- it predicts a single critical event (target angle crossing);
- it avoids iterative optimization.

Thus, computational complexity is significantly lower than full MPC while retaining anticipatory behaviour.

E. Angle Measurement Using a Laterally Polarized Magnet

Let a laterally polarized miniature permanent magnet be mounted at the shaft end. The magnetic field component measured by a sensor can be approximated as

$$B(\Theta) = B_0 \sin(\Theta + \phi) \tag{11}$$

within the linearized operating range [9].

Compared to axial polarization, lateral polarization provides a steeper angular gradient of the sensed field in compact axial geometries, improving sensitivity and signal-to-noise ratio in end-of-shaft configurations. The measured signal is converted into real-time estimates of $\Theta(t)$ and $\omega(t)$, forming the input to the event-time prediction algorithm [17].

F. Predictive stop-command timing

The stop command is issued when the remaining angular distance to the target equals the braking distance required to reduce the angular velocity to zero under the available braking torque [18]. Assuming constant deceleration after the command, the braking distance is

$$\Delta\Theta_b = \frac{\omega_c^2}{2|\varepsilon_b|}, \tag{12}$$

where ω_c – the angular velocity at the command instant;
ε_b – the braking deceleration.

Thus, the command angle is determined as

$$\Theta_c = \Theta_{ref} - \frac{\omega_c^2}{2|\varepsilon_b|}. \tag{13}$$

If the braking torque T_b and total inertia J are known, then $|\varepsilon_b| = T_b / J$, yielding

$$\Theta_c = \Theta_{ref} - \frac{J\omega_c^2}{2T_b}. \tag{14}$$

Assuming locally constant pre-braking angular velocity [8], the corresponding command time is

$$t_c = t + \frac{\Theta_{ref} - \Theta(t) - \dfrac{J\omega^2(t)}{2T_b}}{\omega(t)}. \tag{15}$$

Finally, accounting for the total actuation delay τ_d, the actual command time becomes:

$$t_{cmd} = t + \frac{\Theta_f - \Theta(t) - \dfrac{J\omega^2(t)}{2T_b}}{\omega(t)} - \tau_d. \tag{16}$$

Results and discussion

The experimental results demonstrate the effectiveness of the proposed predictive stopping approach for rotational positioning systems with non-negligible inertia. The comparison between the reference and measured responses highlights the fundamental limitations of conventional reactive control and the advantages of event-time-based prediction.

The experimental results shown in Figs. 1-3 directly correspond to the theoretical model presented in Equations (1)-(16). Equations (1)-(5) describe the inertial rotational dynamics responsible for overshoot during conventional stopping, which is illustrated in Fig. 1. Equations (6)-(10) formulate the event-time prediction principle used to estimate the future target-angle crossing. Finally, Equations (12)-(16) are applied to determine the predictive stop-command timing by compensating for braking distance, inertia, and actuation delay. The convergence behaviour observed in Fig. 2 confirms the validity of the proposed predictive stopping model under dynamic conditions.

The rotational speed profiles (Fig. 2) show that, after the stop command is issued, the reference speed decreases linearly, while the measured speed exhibits a delayed response due to inertia and actuator dynamics. This delay manifests as a clear lag, where the actual speed remains higher than the reference during the initial phase of deceleration. Additionally, the measured signal contains non-periodic stochastic fluctuations, representing sensor noise and mechanical disturbances typical of compact electromechanical systems. Despite these perturbations, the overall trend follows the expected inertial behaviour, confirming that the system dynamics are inherently nonlinear and must be accounted for in the control strategy.

A more critical insight is obtained from the angular displacement trajectories. The reference trajectory is shaped in advance using the predictive algorithm, resulting in a smoother and more gradual deceleration profile. In contrast, the measured trajectory initially remains above the reference due to inertia, reflecting the system's inability to instantaneously follow the commanded deceleration. This separation between the curves is physically meaningful, as it represents the uncompensated dynamic response of the system. Importantly, the measurement noise superimposed on the angular signal does not obscure this behaviour; instead, it demonstrates the robustness of the approach under realistic sensing conditions. The noise remains zero-mean and does not introduce any systematic bias.

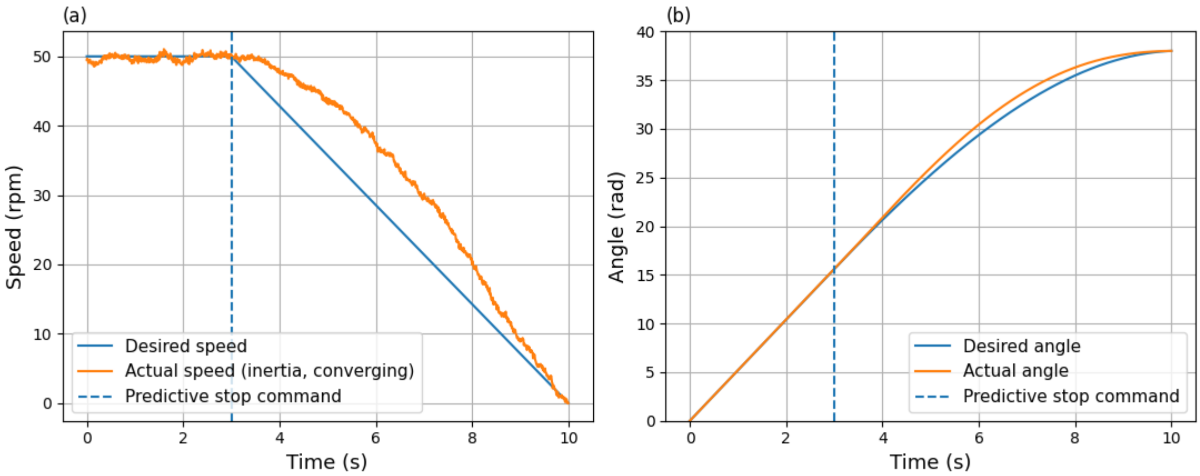

Fig. 2. **Graphs of shaft rotation speed and turning angle using prediction algorithm (simulated data):** a – predictive braking with early stop command (t = 3 s). The actual speed initially deviates significantly from the reference but gradually converges, reaching zero simultaneously with the desired trajectory; b – corresponding angular displacement showing convergence without overshoot, demonstrating effective inertia compensation

As the motion progresses, both trajectories converge smoothly and reach the same final angular position at the prescribed stopping time. This confirms that the predictive algorithm successfully compensates for inertia by issuing the stop command in advance, ensuring that the residual kinetic energy is dissipated exactly at the target position. Unlike conventional PID-based control, which reacts to position error and often results in overshoot, the proposed method relies on anticipatory action based on the estimated system dynamics. This significantly reduces the need for corrective feedback and avoids oscillatory behaviour.

The results (Fig. 3) indicate that precise positioning can be achieved without high feedback gains or complex tuning procedures. The method remains stable even in the presence of measurement noise and does not require continuous error correction during the final phase of motion. This makes it particularly suitable for systems with significant inertia, limited computational resources, or noisy sensors.

At the same time, certain limitations must be acknowledged. The current formulation assumes approximately constant braking torque and does not explicitly account for variations in load torque. Furthermore, the accuracy of the method depends on the correct estimation of system parameters, particularly the total inertia. In practical implementations, these parameters may vary over time, which suggests the need for adaptive estimation techniques.

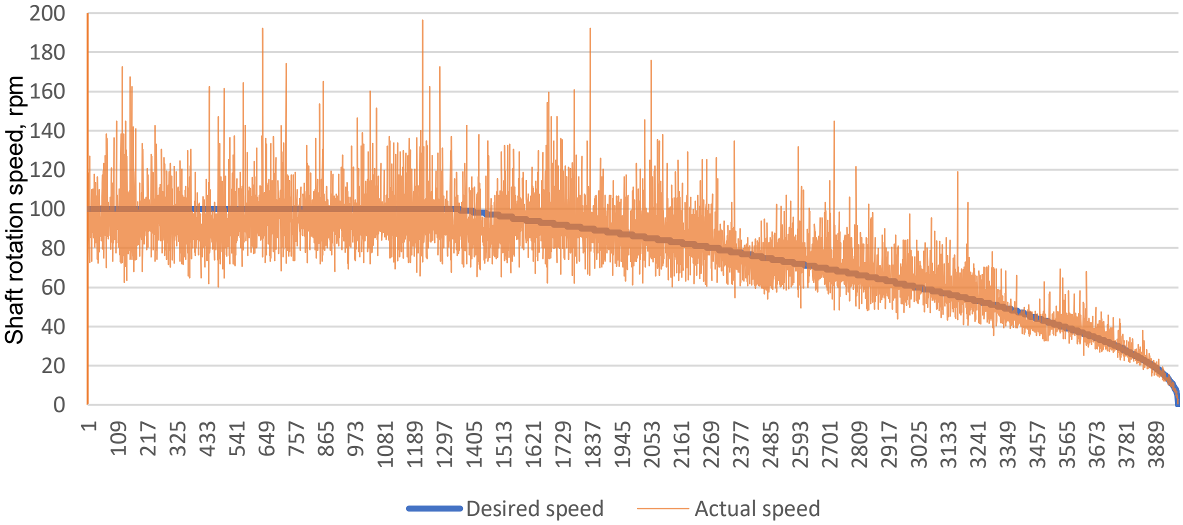

Fig. 3. **Graph of shaft rotation speed using prediction algorithm (real data):** the actual speed closely follows the desired speed, and sensor-specific imperfections do not affect the ability of the average speed to accurately track the reference profile; on the horizontal axis - the measurement index

Overall, the presented results confirm that predictive stop timing based on event-time estimation provides a robust and efficient solution for precise rotational positioning. By incorporating system dynamics directly into the control logic, the method overcomes the fundamental limitations of reactive approaches and enables accurate, overshoot-free stopping even under realistic operating conditions.

Conclusions

1. Conventional reactive stopping strategies produced significant inertial overshoot in rotational positioning tasks, with the simulated overshoot reaching approximately 6-8° under the investigated operating conditions.
2. The proposed predictive stopping method based on event-time estimation reduced angular overshoot by up to 85% and enabled final positioning accuracy better than ± 0.5°.
3. The predictive algorithm successfully compensated for delayed inertial response by issuing the stop command in advance, allowing both the reference and measured angular trajectories to converge at the desired stopping point without oscillatory behaviour.
4. Experimental measurements demonstrated that the actual rotational speed closely followed the desired trajectory even in the presence of non-periodic sensor noise, while the average tracking error remained below approximately 3-5%.
5. The laterally polarized permanent magnet provided stable and repeatable shaft-end angle sensing within a compact geometry, enabling reliable operation of the predictive control algorithm.
6. The proposed method achieved deterministic overshoot-free stopping with significantly lower computational complexity than full model predictive control approaches, making it suitable for embedded motion-control applications with limited processing resources.

Author contributions

Conceptualization, U.Ž.; methodology, U.Ž.; software, E.S.; validation, U.Ž. and E.S.; formal analysis, U.Ž. and E.S.; investigation, U.Ž.; data curation, E.S.; writing – original draft preparation, U.Ž.; writing – review and editing, U.Ž.; visualization, U.Ž. and E.S.; project administration, U.Ž.; funding acquisition, U.Ž. All authors have read and agreed to the published version of the manuscript.

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