

MATHEMATICAL MODELING OF TEXTILE ABRASION IN MARTINDALE TEST

Rolands Breza¹, Uldis Strautins²

¹University of Latvia, Latvia;

²Institute of Mathematics and Computer Science, University of Latvia, Latvia
rolands.breza@gmail.com, uldis.strautins@lu.lv

Abstract. One of the most widely used textile abrasion test standards is the Martindale test where the subject textile is fixed and mechanically rubbed against the abrasive cloth for a prescribed number of cycles and then the quality and durability of the textile can be evaluated based on the frequency and extent of hole formation. The objective of this work is to propose a mathematical model describing the resulting wear which is primarily based on the Reye-Archard-Khrushchov wear law and adapted for this specific purpose. The subject textile and abrasive cloth are modelled as spatially periodic surfaces the thickness of which evolves over time depending on the wear rate at the contact regions. The specific pattern of the movement of the subject textile can be described by a Lissajous trajectory given as a function of time. The resulting system of partial differential equations is discretized and solved numerically, after which the simulation results are compared to the experimental data. In the comparison, it can be observed that certain qualitative properties in the form of wear patterns are present both in the simulations and tests. Furthermore, some of the simulations reveal an emergence of parallel stripes which correspond with real test results and show that using appropriate parameters it is possible to obtain realistic outcomes. Finally, the limitations of the model and possible directions for future development are discussed, including the incorporation of rotation, use of more advanced numerical methods, and finding of exact parameters for recreating test results more precisely.

Keywords: Martindale test, textile abrasion, mathematical model, numerical simulation.

Introduction

Textile abrasion is one of the key parameters that affect service life and quality of fabrics. It is of particular importance for fields that require durability under extreme conditions, such as firefighting, medical, astronaut and military uniforms [1]. In the consumer goods sector, increasing demand for sustainable and high-quality clothing, upholstery fabrics, and other textile products has made the question of abrasion resistance increasingly relevant [2].

To accurately assess textile durability, experimental tests such as Martindale abrasion test are performed according to the international standard ISO 12947 [3]. The test specimen is clamped in a holder and subjected to a specified normal load, then translated across the surface of a standard abrasive cloth following a Lissajous trajectory, which ensures uniform, multidirectional friction. The specimen holder may simultaneously rotate freely about an axis perpendicular to the fabric surface, further imitating real world wear conditions. While these tests are reliable, they are time-consuming and resource-intensive, creating a need for mathematical models that can simulate the abrasion process and predict material behaviour under various conditions as has been done, for example, in I. Dabolina's work [4].

Several studies have analysed experimental abrasion results, yet systematic mathematical models for textile abrasion simulation are practically non-existent. An interesting study by Woldman et al. [5] examines tool wear caused by contact with sand particles and considers several wear mechanisms using finite elements; however, it cannot describe the mutual abrasion between two fabrics. This indicates a potentially empty niche between engineering experiments and mathematical modelling.

The aim of this work is to present a theoretically grounded, mathematically consistent model for textile abrasion in the Martindale test, based on the Reye-Archard-Khrushchov wear law [6], and to implement it numerically. Some initial work for this has been done by one of the authors [7]. The work outlined here builds on those ideas and applies them specifically to this problem. The model is adapted to Martindale test parameters and supplemented with a simulation that allows determination of wear intensity and potential fabric damage formation.

Materials and methods

The fundamental idea of the model is derived from the Reye-Archard-Khrushchov wear law [6, 8]:

$$V(t) = \frac{KWL}{H}, \quad (1)$$

where K – wear coefficient;
 W – applied normal force, N;
 L – sliding distance, m;
 H – hardness coefficient.

Since we are interested in the temporal evolution of wear, differentiating previous equation (1) with respect to time, and assuming that W , H , and K are constant, yield the volumetric wear rate:

$$\tilde{\omega}(t) = \frac{\partial V}{\partial t}(t) = \frac{KWv(t)}{H}, \quad (2)$$

where $v(t) = \partial L / \partial t$ is the instantaneous sliding speed, $\text{m} \cdot \text{s}^{-1}$.

Both the subject textile and the abrasive cloth are modelled as spatially periodic surfaces. The abrasive surface S_{abr} is defined on a domain $\Omega \subset \mathbb{R}^2$, while the subject textile surface \tilde{S}_{sub} is defined on $\tilde{\Omega} \subset \Omega$. At time $t = 0$, the surface heights are given by products of shifted sinusoidal functions:

$$S_{abr}(x, y, 0) = (A_{abr} + \sin(k_{abr}x)) \cdot (B_{abr} + \sin(m_{abr}y)), \quad (3)$$

$$\tilde{S}_{sub}(x, y, 0) = (A_{sub} + \sin(k_{sub}x)) \cdot (B_{sub} + \sin(m_{sub}y)), \quad (4)$$

where $A_{abr}, B_{abr}, A_{sub}, B_{sub}$ – amplitude coefficients for the abrasive and subject textile;
 $k_{abr}, m_{abr}, k_{sub}, m_{sub}$ – frequency coefficients for the abrasive and subject textile.

The resulting product of the sinusoidal representation effectively captures the periodic weave structure observed in real textiles [9].

To ensure that the two surfaces touch without overlapping, the subject surface is inverted and shifted. Specifically, the positioned subject surface is

$$S_{sub}(x, y, t) = -\tilde{S}_{sub}(x, y, t) + \max_{(x,y) \in \tilde{\Omega}} (\tilde{S}_{sub}(x, y, t) + S_{abr}(x, y, t)). \quad (5)$$

Since both surfaces are continuous, they will touch in at least one point. The distance between the two surfaces at any point (x, y) is

$$d(S_{sub}(x, y, t), S_{abr}(x, y, t)) = |S_{sub}(x, y, t) - S_{abr}(x, y, t)|, \quad (6)$$

and the contact region therefore could be defined as

$$\Gamma = \{(x, y) \mid d(S_{sub}, S_{abr}) < \varepsilon\}, \quad (7)$$

where ε – some small positive number.

The contact indicator function will be defined then as

$$\delta(x, y, t) = \begin{cases} 1, & (x, y) \in \Gamma, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Whilst assuming that the normal force W is constant can seem practical for the ease of calculation, we suggest replacing it by taking into consideration the surface topography of the textile as the wear rate depends on whether it moves uphill or downhill. This can be captured using an angle θ between the velocity vector and the surface gradient and a modulation function

$$g(\theta) = \frac{1 + \sqrt{|1 - \cos^2 \theta|}}{2} \in [0, 1]. \quad (9)$$

The complete wear rate for each abrasive surface (with an analogous form for subject textile) takes the form:

$$\omega_{abr}(x, y, t) = (\omega_{min} + (\omega_{max} - \omega_{min})g(\theta)) \cdot v(t) \cdot \frac{K_{abr}}{H_{abr}}, \quad (10)$$

where ω_{min} and ω_{max} – dimensionless constants bounding the directional motion;

K_{abr}, H_{abr} — specific coefficients K and H for abrasive surface.

Thus, the evolution of both surfaces is governed by the system of partial differential equations:

$$\frac{\partial S_{abr}}{\partial t}(x, y, t) = -\omega_{abr}(x, y, t) \cdot \delta(x, y, t), \quad (11)$$

$$\frac{\partial \tilde{S}_{sub}}{\partial t}(x, y, t) = -\omega_{sub}(x, y, t) \cdot \delta(x, y, t) \quad (12)$$

subject to the initial conditions (3) and (4). Note that material accumulation along the edges is neglected in this work; however, it is worthwhile considering such a case by introducing an additional indicator function $\tilde{\delta}(x, y, t)$ that would act on the surfaces with an opposite effect to $\delta(x, y, t)$.

The motion of the subject textile follows a Lissajous curve, which is a combination of two perpendicular harmonic motions:

$$x(t) = A \sin(at + \varphi), y(t) = B \sin(bt), \quad (13)$$

where A, B – amplitudes;

φ – phase shift;

a, b – parameters that control the trajectory shape.

Based on publicly available information from testing instrument manufacturers [10][9] and values used in prior studies, the parameters $a = 15, b = 16, \varphi = \pi/2$ are adopted. It is possible to calculate a complete cycle period, which can be calculated as

$$T_{cycle} = 2\pi \cdot \frac{\text{lcm}(a, b)}{ab} \quad (14)$$

since $\text{gcd}(15, 16) = 1$, the cycle period equals to 2π .

Numerical discretization

The previously discussed system of partial differential equations (11) and (12) with initial conditions (3) and (4) forms an initial value problem, which in this work is discretized using the forward Euler method. The time interval $[0, T]$ is divided into $N_\tau = T/\tau$ uniform steps with step size $\tau > 0$. The spatial domains Ω and $\tilde{\Omega}$ are discretized as Cartesian products of uniform grids with $N_x \times N_y$ and $N_{sub} \times N_{sub}$ points, respectively. The algorithm can be summarized in a form:

$$S_{abr}^{n+1} = S_{abr}^n + \tau f_{abr}^n, n = 0, 1, 2, \dots \quad (15)$$

where $f_{abr}^n = -\omega_{abr}^{(n)}(x, y) \cdot \delta^n(x, y)$.

The formula is analogous for \tilde{S}_{sub}^n . The velocity is approximated by central differences:

$$v_x^n = \frac{x_{n+1} - x_{n-1}}{2\tau}, v_y^n = \frac{y_{n+1} - y_{n-1}}{2\tau} \quad (16)$$

and the surface gradient is computed using central finite differences. At each time step, the subject surface position is mapped onto the abrasive grid via index rounding, and contact regions are identified. The implementation was carried out in MATLAB.

Results and discussion

Numerical experiments were performed with the following baseline parameters unless stated otherwise:

$$A_{sub} = A_{abr} = B_{sub} = B_{abr} = 1; \omega_{min} = 5, \omega_{max} = 20; T = 628; \tau = 0.05;$$

$$L_x = L_y = 55; N_x = N_y = 2500; L_{sub} = 15; N_{sub} = 1000;$$

$$K_{sub} = K_{abr}; \varepsilon = \frac{1}{20} \max_{(x,y) \in \Omega} S_{sub}^n(x, y).$$

During the simulations there were observed 3 noticeable behaviors that came out of this model that were then compared to the experimental data of I. Dabolina:

- *Emergence of parallel stripe patterns.* With parameters $H_{abr} = 10$; $H_{sub} = 1$, and symmetric frequency parameters $k_{abr} = m_{abr} = 4\pi/5$; $k_{sub} = m_{sub} = 8\pi/7$, the simulation at $t = 116.95$ produces a wear pattern exhibiting clearly visible parallel diagonal stripes in Fig. 1. **Simulation** This pattern qualitatively matches the experimental results observed on the 3rd specimen after 14 000 cycles in Fig. 2, where holes also form along parallel diagonal lines. Comparing the resulting picture to that of the experimental photography reveals a convincing visual correspondence of the stripe orientations.

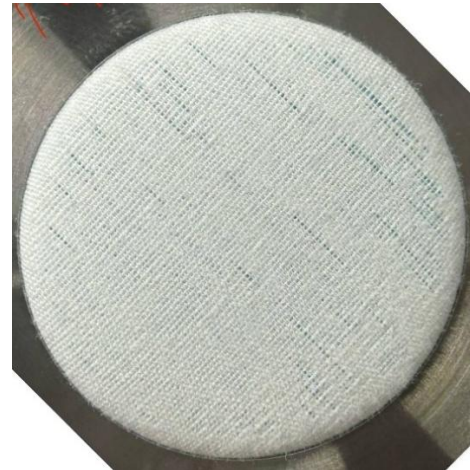
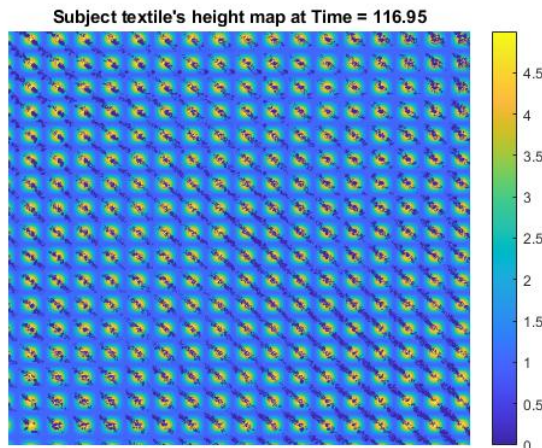


Fig. 1. Simulation with clearly visible stripes

Fig. 2. Parallel wear patterns on experimental sample

- *Non-stripe wear patterns.* When asymmetric frequency parameters are used, e.g. $k_{abr} = 3\pi/10$; $m_{abr} = 4\pi/5$; $k_{sub} = 3\pi/7$; $m_{sub} = 8\pi/7$ with $H_{sub} = 0.01$, $H_{abr} = 1$, the simulation produces a rough, irregular damage pattern without discernible parallel stripes. One such result can be seen in Fig. 3, where the formed holes do not create any parallel structures. Increasing the hardness coefficients while maintaining the same ratios does not alter the spatial arrangement of holes, only the rate of their formation. Based on this it can be stipulated that under this model, the weave symmetry might play a key role in determining wear pattern geometry.

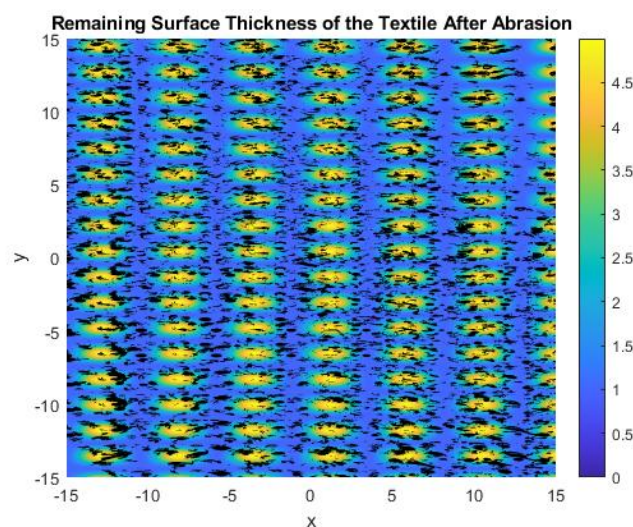


Fig. 3. Simulation showing absence of holes in the form of straight lines

- *Edge concentration of wear.* If in simulation high hardness coefficients ($H_{sub} = 100$; $H_{abr} = 10\,000$) and high-frequency surface parameters are used, then it is

possible to notice another phenomenon arise. The wear as seen in Fig. 4 is concentrated mainly near the edges of the subject textile specimen. This is consistent with experimental observations, where damage typically initiates at the specimen periphery as can be observed in Fig. 5. The Lissajous trajectory produces the highest cumulative contact near the turning points at the boundary of the motion envelope, which physically explains the edge-concentrated wear.

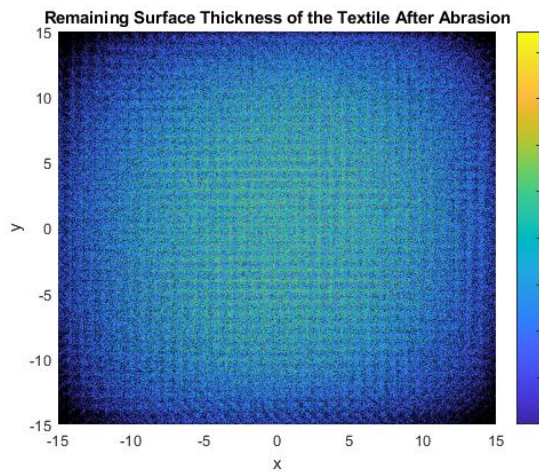


Fig. 4. Concentration of abrasion near corners of simulated textile



Fig. 5. Experimental specimen with rough edges

Conclusions

1. A mathematically consistent model for textile abrasion in the Martindale test has been developed based on the Reye-Archard-Khrushchov wear law, modelling both surfaces as spatially periodic functions which thickness evolves according to contact-driven wear.
2. The model has been discretized using the forward Euler method and implemented in MATLAB, demonstrating the feasibility of numerical simulation of the two-surface abrasion process along a Lissajous trajectory.
3. Qualitative agreement with experimental data has been observed; simulations with symmetric weave parameters produce parallel stripe wear patterns matching those found in laboratory Martindale tests.
4. A hypothesis has been formulated that the symmetry of textile weaving might be linked to the emergence of parallel stripe damage pattern, providing a direction for further research.
5. Future development should include the incorporation of specimen rotation, use of higher-order numerical methods, and experimental validation to calibrate specific model parameters.

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Author contributions

Conceptualization, R.B. and U.S.; methodology, R.B. and U.S.; software, R.B.; validation, R.B.; formal analysis, R.B. and U.S.; investigation, R.B.; writing – original draft preparation, R.B.; writing – review and editing, U.S.; supervision, U.S. All authors have read and agreed to the published version of the manuscript.

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