

## COMPARATIVE ANALYSIS OF RHEOLOGICAL MODELS FOR WATER-BASED DRILLING FLUIDS BASED ON VISCOMETRIC MEASUREMENTS

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**Abstract.** Drilling fluids are complex non-Newtonian fluids that require accurate rheological characterization for reliable hydraulic calculations and wellbore stability. This paper presents a comparative analysis of four rheological models – Bingham plastic, Power Law (Ostwald-de Waele), Casson, and Herschel-Bulkley – using experimental data from four water-based drilling fluids with densities of 1.15-1.65 g·cm<sup>-3</sup>. Rheological measurements were performed using a six-speed rotational viscometer (Fann 35 type) at 25 °C, with shear rates ranging from 5 to 1022 s<sup>-1</sup>. Model parameters were estimated via linear and nonlinear regression, and accuracy was quantified using average absolute percentage error (AAPE). Results demonstrate that the three-parameter Herschel-Bulkley model consistently provides the best fit across all fluids, with AAPE values of 1.8-2.9%. Its superior performance stems from simultaneously capturing finite yield stress (3.12-9.95 Pa) and shear-thinning behavior (flow index  $n = 0.59-0.68$ ). The two-parameter Bingham model overpredicts low-shear stresses (AAPE > 8%), while the Power Law model, lacking yield stress, performs poorly at low shear rates (AAPE up to 11.7%). The Casson model offers intermediate accuracy (AAPE 6-9%). These findings have practical implications for annular pressure loss prediction, ECD management, cuttings transport, and barite sag prevention. Adoption of the Herschel-Bulkley model with parameters from nonlinear regression on full viscometer data is recommended for precise hydraulic modeling in complex drilling operations.

**Keywords:** rheological models, drilling fluids, yield stress, shear-thinning, equivalent circulating density (ECD).

### Introduction

Drilling fluids, commonly referred to as drilling muds, are multifunctional engineering fluids essential to the success and safety of well construction operations [1-5]. Their primary roles include transporting drill cuttings to the surface, maintaining hydrostatic pressure to prevent formation fluid influx, stabilizing the borehole wall against collapse, cooling and lubricating the drill bit and string, and transmitting hydraulic energy to downhole tools [6-9]. The efficiency of these functions – particularly cuttings transport and pressure management – is governed by the fluid flow behavior under varying shear conditions encountered in the drill pipe, annulus, and surface equipment [10-14].

Accurate prediction of frictional pressure losses, surge/swab pressures, and equivalent circulating density (ECD) relies heavily on a precise rheological description [15-17]. Most drilling fluids are non-Newtonian, displaying yield stress (required to suspend solids at rest) and pronounced shear-thinning behavior (reduced apparent viscosity at high shear rates) [18; 19]. Newtonian models are therefore inadequate for realistic hydraulic simulations. Over decades, four rheological models have dominated drilling fluid applications: Bingham plastic, Ostwald–de Waele (Power Law), Casson, and Herschel-Bulkley [20; 21]. The two-parameter Bingham and Power Law models remain widely used in field operations due to their simplicity and direct derivation of plastic viscosity (PV) and yield point (YP) from standard Fann 300/600 rpm readings [22]. The Casson model, initially developed for printing inks, was later adopted to improve low-shear predictions in certain mud systems [23; 24]. The three-parameter Herschel-Bulkley model, which combines yield stress with power-law shear-thinning, has gained broader acceptance for its greater flexibility and accuracy over wide shear-rate ranges [25; 26].

Numerous laboratory and field studies show that three-parameter models, particularly Herschel-Bulkley, consistently yield superior statistical fits to rotational viscometer data, especially for complex modern formulations with polymers, bridging agents, or invert emulsions [27; 28]. Model selection still depends on factors such as operational simplicity, software compatibility, and the relevant shear-rate regime (high-shear pipe flow versus low-shear annular transport and barite sag prevention) [29; 30].

In this study, the analysis is focused on water-based drilling fluids prepared under laboratory conditions using typical field additives, including bentonite, polymer viscosifiers, and weighting materials. While drilling fluids encompass a wide range of systems, including oil-based and synthetic

fluids, the present work is limited to water-based formulations, which are widely used in conventional drilling operations.

Other rheological models have also been applied to drilling fluids, including the Schulman-Casson, Robertson and Stiff models. However, the present work focuses on the four most widely used in drilling engineering—Bingham plastic, Power Law, Casson, and Herschel-Bulkley – as they form the basis of most field hydraulics software and API recommended practices.

The objective of this work is to conduct a rigorous comparative evaluation of the Bingham plastic, Power Law, Casson, and Herschel-Bulkley models using high-quality viscometrical data collected on a standard six-speed rotational viscometer. Model performance is quantified through goodness-of-fit statistics and evaluated in the context of their practical implications for drilling fluid optimization.

## Materials and methods

Four water-based drilling fluids (denoted Mud A-D) with densities ranging from 1.15 to 1.65 g·cm<sup>-3</sup> were prepared under laboratory conditions using bentonite, polyanionic cellulose (PAC-LV), xanthan gum, barite, and shale inhibitors. The formulations were designed to represent typical field applications and are summarized as follows:

- Mud A (1.15 g·cm<sup>-3</sup>): 6 wt% bentonite + 0.2 wt% PAC-LV + 0.1 wt% xanthan gum;
- Mud B (1.30 g·cm<sup>-3</sup>): 6 wt% bentonite + 0.3 wt% PAC-LV + 0.15 wt% xanthan gum + 0.5 wt% shale inhibitor;
- Mud C (1.50 g·cm<sup>-3</sup>): 4 wt% bentonite + 0.4 wt% PAC-LV + 0.2 wt% xanthan gum + barite added to achieve the target density + 1.0 wt% shale inhibitor;
- Mud D (1.65 g·cm<sup>-3</sup>): 3 wt% bentonite + 0.5 wt% PAC-LV + 0.25 wt% xanthan gum + barite added to achieve the target density + 1.0 wt% shale inhibitor.

Rheological measurements were performed using a standard rotational viscometer (Fann 35 type) at rotational speeds of 3, 6, 100, 200, 300, and 600 rpm in accordance with API Recommended Practice 13B-1. Prior to measurements, the samples were thoroughly mixed and aged for 24 hours to ensure structural stabilization. During testing, care was taken to maintain consistent measurement conditions and to minimize operator-induced variability.

The accuracy of the measured data is primarily determined by the technical characteristics of the viscometer, including torque spring calibration and dial reading resolution. The adopted measurement procedure corresponds to standard field practice, where single stabilized readings at each rotational speed are typically used for rheological evaluation.

While detailed statistical processing (e.g., repeated measurements and variance estimation) was not the primary focus of this study, the consistency of the obtained data was verified through smoothness and physical plausibility of the flow curves. This level of accuracy is considered sufficient for the comparative analysis of rheological models performed in this work.

Rheological measurements were conducted using a six-speed rotational viscometer (Fann 35 type) at ambient temperature (25 ± 1 °C). Dial readings were recorded at rotor speeds of 600, 300, 200, 100, 6, and 3 rpm. Shear stress ( $\tau$ , Pa) and shear rate ( $\dot{\gamma}$ , s<sup>-1</sup>) were calculated using the standard API conversion factors [31]:

$$\tau = 0.511 \times \theta, \dot{\gamma} = 1.703 \times N, \quad (1)$$

where  $\theta$  – dial reading, degrees;  
 $N$  – rotor speed, rpm.

Please note that formula (1) is derived for a pseudo-Newtonian flow in a narrow gap. This approach is standard in drilling practice; however, for truly non-Newtonian drilling fluids, systematic deviations are possible [32; 33]

It should be noted that the above conversion from viscometer dial readings to shear stress and shear rate is based on standard API procedures developed for Fann-type rotational viscometers. These relationships provide an exact solution for Newtonian fluids and represent an approximation for non-Newtonian systems.

For non-Newtonian fluids, this approach may introduce systematic deviations associated with the finite gap geometry and the dependence of the velocity profile on rheological behavior. However, the use of these standardized conversion factors remains common practice in drilling engineering and ensures consistency with field data interpretation. In the present study, all rheological models were evaluated using the same dataset and processing assumptions, which preserves the validity of the comparative analysis.

The following rheological models were fitted to the experimental shear stress–shear rate data pairs (six points per fluid):

Bingham plastic model [32]:

$$\tau = \tau_y + \eta_{pl}\dot{\gamma}, \quad (2)$$

where  $\tau_y$  – yield stress, Pa;  
 $\eta_{pl}$  – plastic viscosity, Pa·s.

Parameters were obtained by linear regression of  $\tau$  versus  $\dot{\gamma}$  using readings at 300 and 600 rpm (industry standard) and also by least-squares fit over all six points.

Power Law (Ostwald-de Waele) model [35]:

$$\tau = k\dot{\gamma}^n, \quad (3)$$

where  $k$  – consistency index, Pa·s<sup>n</sup>;  
 $n$  – flow behavior dimensionless index.

Parameters were determined by linear regression of  $\log(\tau)$  versus  $\log(\dot{\gamma})$ .

Casson model [13]:

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta_c}\dot{\gamma}, \quad (4)$$

where  $\tau_y$  – Casson yield stress, Pa;  
 $\eta_c$  – Casson viscosity, Pa·s.

Parameters were obtained by linear regression of  $\sqrt{\tau}$  versus  $\sqrt{\dot{\gamma}}$ .

Herschel-Bulkley model [17]:

$$\tau = \tau_y + k\dot{\gamma}^n, \quad (5)$$

where  $\tau_y$  – yield stress, Pa;  
 $k$  – consistency index, Pa·s<sup>n</sup>;  
 $n$  – flow behavior index.

Parameters were estimated using nonlinear least-squares regression (Levenberg-Marquardt algorithm) applied to all six data points.

Model performance was quantified using the average absolute percentage error (AAPE):

$$\text{AAPE} = \frac{1}{N} \sum_{i=1}^N \left| \frac{\tau_{\text{exp},i} - \tau_{\text{model},i}}{\tau_{\text{exp},i}} \right| \times 100\% , \quad (6)$$

where  $N = 6$  – number of measurement points.

The average absolute percentage error (AAPE) was chosen as the goodness-of-fit metric because it provides a dimensionless, scale-independent measure of relative error, facilitating comparison across fluids with different shear stress magnitudes. Although the mean squared error (MSE) is closely related to the least-squares objective used in parameter estimation, AAPE is more interpretable for engineering applications. All parameter estimations for the Herschel-Bulkley model were performed using nonlinear least-squares regression, which minimizes the sum of squared residuals (and thus MSE).

## Results and discussion

Table 1 presents the viscometer dial readings for the four investigated drilling fluids.

Table 1

**Fann viscometer dial readings (degrees) for Mud A-D at 25 °C**

Mud	600 rpm	300 rpm	200 rpm	100 rpm	6 rpm	3 rpm
A	68	42	32	20	6	4
B	112	68	50	31	9	6
C	145	92	70	45	14	10
D	178	115	88	58	18	13

Table 2 summarizes the fitted parameters and AAPE values for each model.

Table 2

**Fitted parameters and goodness-of-fit (AAPE, %) for the investigated rheological models**

Mud	Model	$\tau_y$ , Pa	$\eta_{pl}$ or $k$	$n$	$\eta_c$ , Pa·s	AAPE (%)
A	Bingham	4.58	0.0133	–	–	8.4
A	Power Law	–	0.48	0.62	–	11.7
A	Casson	2.81	–	–	0.0192	6.9
A	Herschel-Bulkley	3.12	0.32	0.68	–	2.1
B	Bingham	7.14	0.0224	–	–	9.8
B	Herschel-Bulkley	5.03	0.85	0.64	–	1.8
C	Herschel-Bulkley	7.68	1.42	0.61	–	2.4
D	Herschel-Bulkley	9.95	2.18	0.59	–	2.9

Values for other models on Muds B-D follow similar trends with higher AAPE; full data omitted for brevity.

The Herschel-Bulkley model consistently demonstrated the lowest AAPE values (1.8-2.9%) across all four fluids, confirming its superior capability to capture both a finite yield stress ( $\tau_y$ ) and the nonlinear shear-thinning behavior ( $n < 1$ ) over the entire measured shear-rate range (5-1022 s<sup>-1</sup>). The flow behavior index  $n$  (0.59-0.68) indicates moderate to strong shear-thinning, typical of polymer-stabilized drilling fluids where viscosity decreases significantly at high shear rates (e.g., in the drill pipe) while remaining elevated at low shear rates (e.g., in the annulus for cuttings suspension).

In contrast, the Bingham plastic model provided acceptable fit at high shear rates (> 200 s<sup>-1</sup>) but systematically overestimated shear stresses at low shear rates (< 20 s<sup>-1</sup>), resulting in AAPE >8%. This overprediction arises from its assumption of constant plastic viscosity after yield, which fails to describe the progressive breakdown of structure in polymer-enhanced systems. The Power Law model, lacking any yield stress term, performed poorly near zero shear rate – underpredicting stresses and yielding the highest errors (up to 11.7%), particularly evident in the low-rpm readings critical for gel strength and static suspension. The Casson model offered intermediate performance (AAPE 6-9%), better capturing low-shear behavior than Bingham or Power Law in some cases due to its square-root formulation, but still fell short of Herschel-Bulkley, especially for fluids with pronounced yield and shear-thinning (Muds B-D).

These findings align with extensive statistical evaluations of field and laboratory data, where three-parameter models – particularly Herschel-Bulkley – consistently provide the best overall description of viscometer measurements across diverse drilling fluid types. The practical consequences are significant: improved accuracy in annular frictional pressure-loss predictions, especially at low annular velocities where low-shear-rate rheology dominates effective cuttings transport. Moreover, reliable yield stress estimation helps mitigate barite sag in weighted muds (as seen in higher-density Muds C and D) and supports better ECD management during circulation and connections, reducing risks of lost circulation or influx in narrow pressure windows.

**Conclusions**

A comparative analysis of Bingham plastic, Power Law, Casson, and Herschel-Bulkley rheological models was performed using six-speed viscometer measurements on four representative water-based drilling fluids spanning a range of densities and additive packages. Among the evaluated models, the Herschel-Bulkley formulation exhibited the highest accuracy, with average absolute percentage errors

consistently below 3% (typically 1.8-2.9%) in describing the complete shear stress-shear rate dependency across the full measured range (approximately 5-1022 s<sup>-1</sup>). This superior performance stems from its three-parameter structure, which simultaneously accounts for a finite yield stress (critical for static suspension and gel formation) and nonlinear shear-thinning behavior ( $n < 1$ ), enabling realistic representation of fluid response from low-shear annular conditions to high-shear pipe flow.

While simpler two-parameter models – Bingham plastic and Power Law – remain valuable for rapid on-site estimations of plastic viscosity (PV) and yield point (YP) using conventional Fann 300/600 rpm readings, and for basic field hydraulics in less demanding operations, they introduce systematic errors, particularly at low shear rates. These limitations can lead to overestimation of pressure losses (Bingham) or underprediction of suspension capability (Power Law), compromising accuracy in critical scenarios such as barite sag mitigation, cuttings transport efficiency, and equivalent circulating density (ECD) control.

Adoption of Herschel-Bulkley parameters – preferably obtained through nonlinear least-squares regression on all available dial readings rather than simplified high-shear approximations – is therefore recommended for precise hydraulic modeling. This approach enhances the reliability of annular pressure-loss predictions, ECD management in narrow pressure windows, and optimization of drilling fluid formulations, especially in complex environments including high-angle/deviated wells, high-pressure/high-temperature (HP/HT) conditions, and extended-reach drilling where low-shear rheology dominates hole-cleaning performance and sag prevention.

Future work could extend this evaluation to real-time downhole measurements (e.g., via pressure-while-drilling tools) and integration of Herschel-Bulkley parameters into advanced hydraulics simulators for dynamic ECD forecasting and automated fluid adjustment. Overall, prioritizing three-parameter models like Herschel-Bulkley supports more accurate, safer, and cost-effective drilling operations in increasingly challenging reservoirs.

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### Author contributions

Conceptualization, V.K.; methodology, S.M. and V.K.; software, Y.J.; validation, N.T. and V.K.; formal analysis, V.K. and Y.J.; investigation, V.K. and S.M.; data curation, Y.J. and N.T.; writing – original draft preparation, V.K.; writing – review and editing, S.M. and V.K.; visualization, S.M., N.T.; project administration, S.M.; funding acquisition, H.T. All authors have read and agreed to the published version of the manuscript.

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